

Stability, chaos and entrapment of stars in very wide pairs

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ABSTRACT

The relative motion of stars and other celestial objects in very wide pairs, separated by distances of the order of 1 pc, is strongly influenced by the tidal gravitational potential of the Galaxy. The Coriolis component of the horizontal tidal force in the rotating reference frame tends to disrupt such marginally bound pairs. However, even extremely wide pairs of bodies can be bound over intervals of time comparable to the Hubble time, under appropriate initial conditions. Here we show that for arbitrary chosen initial coordinates of a pair of stars, there exists a volume of the space of initial velocity components where the orbits remain bound in the planar tidal field for longer than 10 Gyr, even though the initial separation is well outside the Jacobi radius. The boundary of this phase space of stable orbits is fractal, and the motion at the boundary conditions is clearly chaotic. We found that the pairs may remain confined for several Gyr, and then suddenly disintegrate due to a particularly close rendezvous. By reversing such long-term stable orbits, we find that entrapment of unrelated stars into wide pairs is possible, but should be quite rare. Careful analysis of precision astrometry surveys revealed that extremely wide pairs of stars are present in significant numbers in the Galaxy. These results are expected to help in discriminating the cases of genuine binarity and chance entrapment, and to make inroads in testing the limits of Newtonian gravitation.

Key words: chaos – celestial mechanics – binaries: general – stars: kinematics and dynamics.

1 INTRODUCTION

The Galactic tidal force becomes comparable to the force of gravitational attraction of widely separated double stars at distances close to the Jacobi radius, which approximately equals 1.8 pc for twin stars of solar mass. Beyond the Jacobi radius, the planar component of the tidal force may cause a relatively rapid disruption of a wide pair. Stars emerging from disrupted binary systems or stellar associations are stretched out by the Galactic potential into narrow streams in the direction of Galactic rotation, a process which is well described by the epicycle approximation (Makarov, Olling & Teuben 2004). This does not mean that all pairs of stars separated by distances greater than the Jacobi radius will be disrupted by the tidal force. The Coriolis component of force acting on a pair of stars in the corotating reference frame centred on the primary (i.e. larger mass) companion is proportional to the instantaneous relative velocity of the secondary companion (Jiang & Tremaine 2010). Under appropriate initial conditions, the Coriolis force can confine the relative motion of two stars to a finite volume of the phase space, even if their mutual distance is far greater than the Jacobi radius. The resulting trajectories are similar to regular orbits, in that they have a periodic character and the separation never exceeds a certain limit within the Hubble time.

In this Letter, we present and discuss the results of a large number of numerical integration of pairs of stars in the rotating, non-inertial reference frame, performed with the J. Chambers' *MERCURY* code, appropriately modified for this case (see Appendix A). The aim of these numerical simulations is to demonstrate that a certain area of the phase space of initial parameters exists well outside the Jacobi radius, where the resulting orbits are regular, bound and stable. By retracing long-term stable orbits at the boundary of the stable area, we prove that accidental capture of two passing stars into a stable double system is possible in principle.

2 STABILITY OF EXTREMELY WIDE DOUBLE STARS

Fig. 1 shows an example of a stable orbit for a hypothetical pair of stars with a total mass of $1.72 M_{\odot}$, travelling on a circular orbit at the orbital radius of the Sun around the Galaxy. The primary star is at the origin of the reference frame ($x = 0$, $y = 0$), and the initial phase-space parameters of the companion are $x_0 = 0$, $y_0 = -4 \times 10^5$ au, $\dot{x}_0 = 4.8 \times 10^{-5}$ au d⁻¹, $\dot{y}_0 = 0$. The companion, moving in shifting double loops around the primary for 10 Gyr (which was our integration time), never wanders away by more than the initial separation. The Jacobi radius for this case is equal to 3.55×10^5 au, assuming the values for the Oort constants determined from the *Hipparcos* data (Makarov & Murphy 2007). Throughout this Letter, the x -axis is directed towards the Galactic Centre, the y -axis

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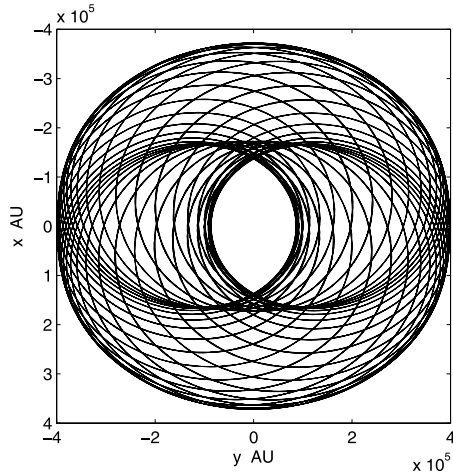


Figure 1. Trajectory of a $0.72 M_{\odot}$ companion around a $1 M_{\odot}$ star in the plane of the Galaxy, integrated over 10 Gyr. The x -axis in this corotating reference frame is pointing towards the Galactic Centre and the y -axis is in the direction of Galactic rotation. The primary component is at the coordinate origin.

towards the direction of Galactic rotation, and our computations are restricted to the planar case, as explained in Appendix A.

Such long-term stable orbits at extreme separations are only possible when the orbital angular momentum is opposite to the Galactic rotation, i.e. when the relative motion is retrograde. As seen from the North Galactic Pole, the orbital motion is always anticlockwise in stable pairs. This important property has been deduced for the classical restricted three-body problem (Hénon 1970) and, more recently, for regular orbits of stars around open clusters in the Galactic tidal potential (Fukushige & Heggie 2000). A corotating pair of stars beyond the Jacobi radius is disrupted by the Coriolis acceleration well before it completes a full revolution in the Galaxy (approximately 230 Myr). Thus, the chances of survival of very wide double systems are defined by initial conditions, barring external perturbations from other field stars. By integrating numerous trajectories at a step of 0.1 au d^{-1} in \dot{x}_0 and \dot{y}_0 for the previously used initial coordinates $x_0 = 0$, $y_0 = -4 \times 10^5 \text{ au}$, we found that the area of stable orbits is approximately triangular in shape defined by the vertices at $(\dot{x}_0, \dot{y}_0) = \{(-4.74, 0), (+4.74, 0), (0, 7.0)\} \times 10^{-5} \text{ au d}^{-1}$. Most of the trajectories with initial velocities inside this triangle are bound in the long term (over 10 Gyr or longer). Most of the trajectories outside this triangle are not bound, with the distance rapidly increasing beyond the initial separation.

3 CHAOS AND ENTRAPMENT

The boundary of the phase space of stable orbits assisted by the Galactic tidal field is fractal, as established numerically by performing integrations at a small step in the initial conditions. In the vicinity of this boundary, a tiny change in the initial velocity, for example, may alter the fate of the stellar pair, resulting in orbits that are bound for longer than 10 Gyr, or ones that fall apart within 100 Myr. Fig. 2 shows the survival time of pairs with the initial coordinates at $x_0 = 0$, $y_0 = -4 \times 10^5 \text{ au}$, and initial velocities varied by small amounts around $\dot{x}_0 = 4.7394 \times 10^{-5} \text{ au d}^{-1}$, $\dot{y}_0 = 0$. Sudden changes in survival time take place on scales as small as 0.17 mm s^{-1} in either dimension of the velocity space. In a separate experiment, we obtained a stable orbit over 10 Gyr at $\dot{x}_0 = 4.7398 \times 10^{-5} \text{ au d}^{-1}$, an orbit which disintegrated in 3.26 Gyr at $\dot{x}_0 = 4.73981 \times 10^{-5} \text{ au d}^{-1}$, and, most remarkably, an orbit which

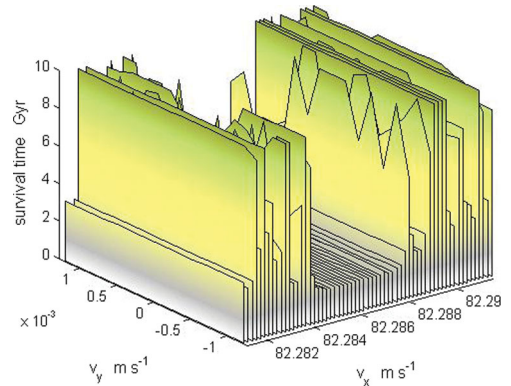


Figure 2. Survival time of pairs of stars initially separated by 400 000 au computed with a step of 0.17 mm s^{-1} in initial velocity components. A tiny variation in phase-space parameters may cause a drastic change in stability at this transitional region separating bound and unbound trajectories.

fell apart after 9 Gyr at $\dot{x}_0 = 4.7398000004 \times 10^{-5} \text{ au d}^{-1}$. The machine precision of these computations is approximately one part in 10^{15} .

The motion of stars with initial parameters at the boundary of the space of stable orbits is strongly chaotic. Using the method to estimate the Lyapunov time suggested for the outer Solar system (Hayes 2007, 2008), two sibling trajectories were integrated at $x_0 = 0$, $y_0 = -4 \times 10^5 \text{ au}$, $\dot{x}_0 = 4.7394 \times 10^{-5} \text{ au d}^{-1}$, $\dot{y}_0 = 0$ by changing the initial value \dot{x}_0 by one part in 10^{11} . The spatial separation between the siblings grew exponentially over the first few Gyr, with an overlaid oscillation of 0.72 Gyr period. The estimated Lyapunov time was 0.56 Gyr, which is approximately equal to the duration of two Galactic revolutions. Both orbits were found stable over 10 Gyr.

The long-term stable orbit at $\dot{x}_0 = 4.7398000004 \times 10^{-5} \text{ au d}^{-1}$, discussed above, is of special interest for the problem of capture of random passing stars into double systems. This pair was bound for 9 Gyr, after which it suddenly fell apart when the secondary component wandered too close to the primary and was accelerated by its gravitational pull. Once the pair is torn apart, the distance between the components begins to grow rapidly. We can construct a long-term stable pair by taking any of the points on the outgoing trajectory, reversing the time and performing a reflection in the four-dimensional phase space. The reflection is necessary, because only retrograde trajectories are stable. A stable trajectory obtained this way for a pair of initially unbound stars is shown in Fig. 3. A chance fly-by star was decelerated by the tidal force, went through a close periastron where it lost more kinetic energy and started to orbit the primary on the characteristic double loops we have seen in Fig. 1. After ~ 8 Gyr of quasi-regular motion, the companion wandered too close to the primary and was ejected again. Obviously, the original trajectory, which was bound for ~ 9 Gyr, could not be exactly reproduced in this experiment. The chaotic motion and the fractal character of the boundary between long-term and short-term trajectories make the machine precision (about 16 significant digits) of our calculations insufficient to match two orbits over such a long time. Thus, accidental entrapment of passing stars into stable pairs is possible in principle, but should occur extremely seldom because of the small area of suitable phase-space parameters. Furthermore, any perturbation from a third passing body will drastically change the fate of the system, in most cases resulting in a relatively rapid breakup. Occasionally, the disturbance from a chance fly-by will drive an initially unstable pair into the space of stable trajectories, but the probability of such events at separations above the Jacobi

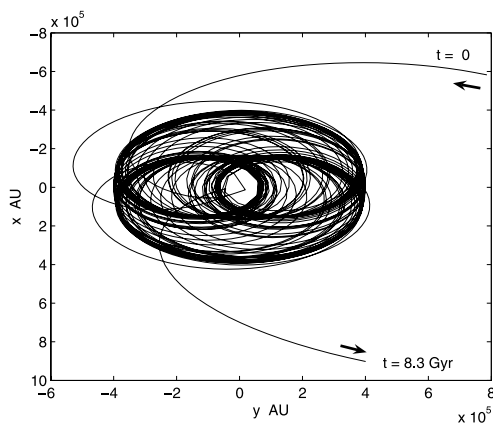


Figure 3. Trajectory of a $0.72 M_{\odot}$ companion around a $1 M_{\odot}$ star in the plane of the Galaxy, entrapped into a bound pair for 8.3 Gyr by the Galactic tidal force. Both the capture and the ejection of the companion were achieved through gravitational manoeuvres in the vicinity of the primary component.

radius is very small. This entrapment mechanism is similar to the chaos-assisted capture of irregular moons by the Sun–Jupiter system, which can only happen in the thin layer of chaotic motion separating the regions of scattering and stability (Astakhov et al. 2003).

4 DISCUSSION

It follows from these considerations that the entrapment of random stars into extremely wide systems has low probability and that such associations should be short-lived. Yet, a growing body of observational evidence indicates that pairs of stars with separations up to 0.1–1 pc are abundant even in the close neighbourhood of the Sun (Makarov, Zacharias & Hennessy 2008; Caballero 2009; Shaya & Olling 2011). Some of these pairs may be the remnants of disintegrating open clusters or OB associations (Kouwenhoven et al. 2010), but not all of the components are sufficiently young. If verified by accurate astrometric observations, the existence of only a few pairs of stars with separations greater than the Jacobi radius in the solar neighbourhood will challenge the classical theory of gravitation in the domain of weak forces. Any modification of Newtonian dynamics in the weak-field regime that boosts the gravitational acceleration at large distances will drastically increase both the probability of entrapment of random pairs of field stars and the survival time of very wide binaries. Even the nearest known stellar system to us, the Alpha Centauri AB and the remote Proxima Centauri companion, may pose a problem for Newtonian dynamics because of the relatively large difference in velocity (Anosova et al. 1994; Wertheimer & Laughlin 2006). Beech (2009) suggested that the apparent stability of this triple system may provide a crucial test for the MOND (Milgrom 1983), or the theory of a coupling between the scalar curvature and the matter Lagrangian density (Bertolami et al. 2007), where an additional force can widen the allowable limit on the orbital acceleration. For the case studied in this Letter ($M_{\text{tot}} = 1.72 M_{\odot}$, $y_0 = -4 \times 10^5$ au), the Newtonian gravitational acceleration is much smaller than the threshold acceleration of MOND ($a_0 \approx 1.2 \times 10^{-10} \text{ m s}^{-2}$). Using the asymptotic expression for acceleration of the companion star, $a = \sqrt{G M_{\text{tot}} a_0} r^{-1}$, we estimate the distance of the Lagrange points to be

$$r_J = \left(\frac{\sqrt{G M_{\text{tot}} a_0}}{4A\Omega} \right)^{1/2}, \quad (1)$$

which for this pair of stars is $r_J = 2.2 \times 10^6$ au, or 10.5 pc. The Jacobi radius in MOND is 6.2 times larger than the Newtonian Jacobi radius, and the area of stable orbits is roughly 38 times greater in the (x, y) plane. Very accurate radial velocity measurements of Proxima Cen would be required for a conclusive test (Beech 2011). Very wide pairs of nearby stars, due to even stricter criteria of survival, may provide the most sensitive experimental data for testing the theories of gravitation in the weak-field regime.

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APPENDIX A: GALACTIC TIDAL FORCE

The planar components of the Galactic tidal acceleration are accurately approximated by the epicycle model for all objects moving on nearly circular orbits at the solar radius (King, Gilmore & van der Kruit 1990; Makarov et al. 2004; Jiang & Tremaine 2010):

$$\begin{aligned} \ddot{x} &= 4A\Omega x - 2\Omega\dot{y}, \\ \ddot{y} &= 2\Omega\dot{x}, \end{aligned}$$

where A is the Oort’s constant, and Ω is the rotation frequency. The vertical component of acceleration, to a good approximation for small vertical velocities at the plane, is harmonic (Makarov et al. 2004) and, therefore, tends to compress very wide binaries. It is sufficient to consider the stability problem restricted to the planar case, ignoring the vertical dimension. All integrations were performed with the well-tested MERCURY code (Chambers & Migliorini 1997) by adding the tidal acceleration components in the user-defined external acceleration subroutine. In the units adopted in the MERCURY code, the assumed parameters were $4A\Omega = 1.19 \times 10^{-20} \text{ d}^{-2}$ and $2\Omega = 1.5 \times 10^{-10} \text{ d}^{-1}$. We used the Bulirsch–Stoer option of integration and set the integration step to 200 d. The companion was considered ejected when the distance from the primary exceeded 900 000 au.

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